

Causal Inference Methods and Case Studies

STAT24630

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Lecture 13

Topic: Matching methods

- Outcome regression V.S. Matching
- Find matched sets
 - Matching metrics and algorithms
 - Check covariate balancing
- Estimate ATT after matching
 - Bias adjustment
- Textbook Chapters 14.1-14.2 & 15 & 18

Causal estimand

- If we treat the units as sampled from a population
 - Population average treatment effect: **PATE = ATE** = $\mathbb{E}(Y_i(1) - Y_i(0))$
 - Average treatment effect for the treated: **PATT = ATT** = $\mathbb{E}(Y_i(1) - Y_i(0) \mid W_i = 1)$
 - Average treatment effect for the control: **ATC** = $\mathbb{E}(Y_i(1) - Y_i(0) \mid W_i = 0)$

$$ATE = P(W_i = 1) \times ATT + P(W_i = 0) \times ATC$$

- In randomized experiments, ATE is equivalent to ATT, because treatment and control groups are comparable in expectation
- In observational studies, we can be interested in ATT
 - Many dataset can have a modest number of treated units, but a relatively large pool of possible controls
 - Treated units are more well defined
 - Control units may include units that never have a chance to receive treatment

Outcome regression estimator

- The outcome regression estimator is the same as in conditional randomized experiment
- Under unconfoundedness assumption

$$\tau = \mathbb{E} \left(\mathbb{E}(Y_i^{\text{obs}} | \mathbf{X}_i, W_i = 1) - \mathbb{E}(Y_i^{\text{obs}} | \mathbf{X}_i, W_i = 0) \right)$$

- Define the conditional expectations


$$\mu_w(\mathbf{x}) = \mathbb{E}(Y_i^{\text{obs}} | \mathbf{X}_i = \mathbf{x}, W_i = w) = \mathbb{E}(Y_i(w) | \mathbf{X}_i = \mathbf{x})$$

- We can estimate the conditional expectations via a regression model and obtain $\hat{\mu}_w(\mathbf{x})$
 - Run a single regression model on all data
 - Regress Y_i^{obs} on \mathbf{X}_i on the treated units and control units separately

- Estimator for the ATE: implement unobserved potential outcome by regression estimates

$$\hat{\tau}_{\text{reg}} = \frac{1}{N} \left\{ \sum_{i=1}^N W_i \left(Y_i^{\text{obs}} - \hat{\mu}_0(\mathbf{X}_i) \right) + (1 - W_i) \left(\hat{\mu}_1(\mathbf{X}_i) - Y_i^{\text{obs}} \right) \right\}$$

model assumptions
on the potential
outcomes



Regression estimator V.S. Matching

- Estimator for the ATT from regression

$$\hat{\tau}_{\text{reg}} = \frac{1}{N_t} \sum_{i=1}^N W_i \left(Y_i^{\text{obs}} - \hat{\mu}_0(\mathbf{X}_i) \right)$$

- Model-based imputation of unobserved potential outcomes
- Drawbacks:
 - biased imputation if model is wrong
 - If the imbalance of the covariates between the two groups is large, the model-based results heavily relies on extrapolation in the region with little overlap, which is sensitive to the model specification assumption
- Matching: nonparametric imputation

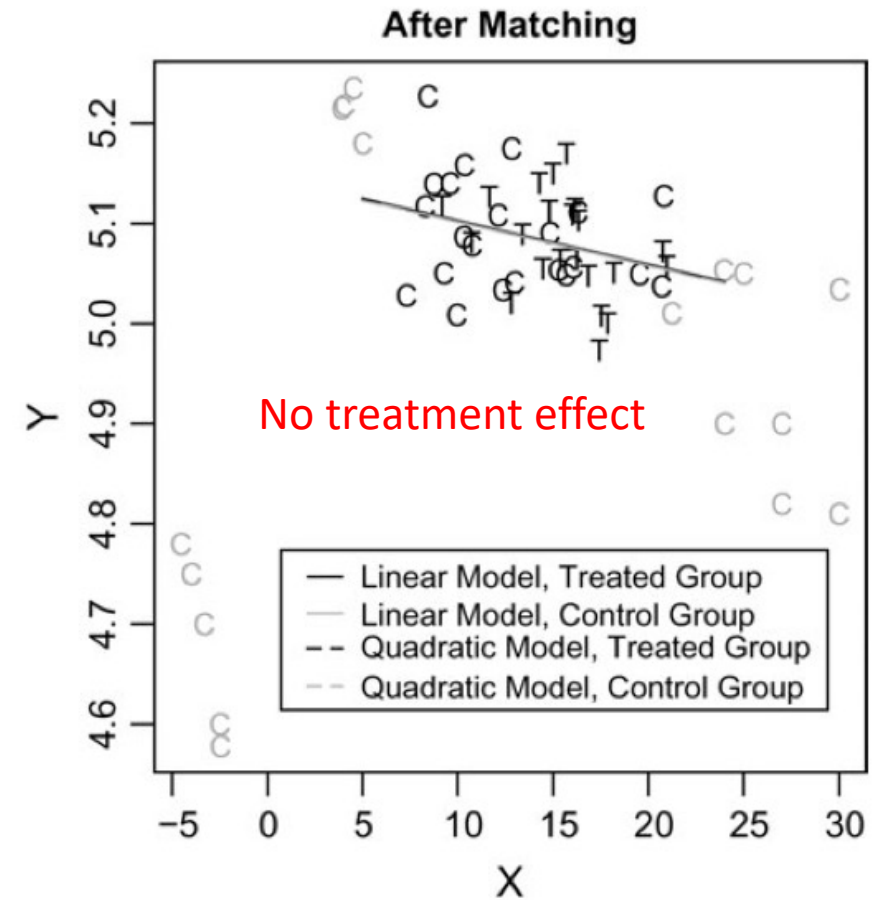
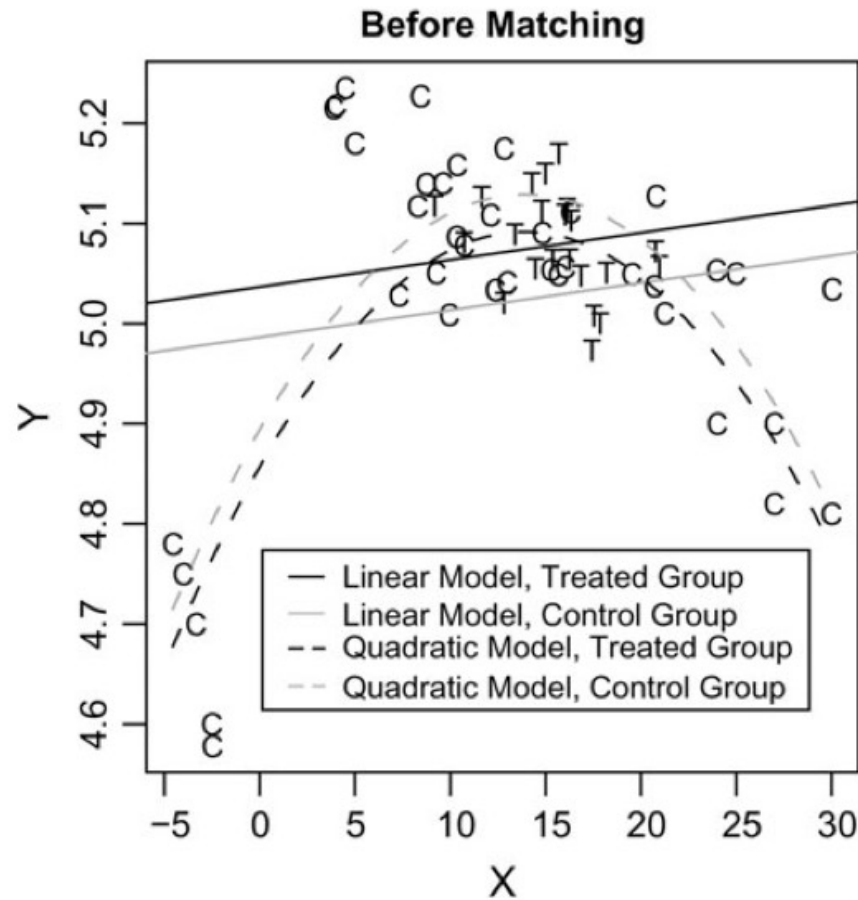
$$\hat{\tau}_{\text{match}} = \frac{1}{N_t} \sum_{i=1}^N W_i \left(Y_i^{\text{obs}} - \frac{1}{|\mathcal{M}_i^c|} \sum_{i' \in \mathcal{M}_i^c} Y_{i'}^{\text{obs}} \right)$$

- \mathcal{M}_i^c : matched set of controls for treated unit i

A simulation data example

[Matching as nonparametric preprocessing for reducing model dependence in parametric causal inference.
Political analysis, 2007]

- Linear regression: positive treatment effect
- Quadratic regression: negative treatment effect
- Both are wrong!!



- At the two extreme tails of X , there are no treatment units at all

How to find matched sets?

- Matching with replacement v.s. matching without replacement
 - Whether we restrict each control to match with at most one treated unit or not
 - Matching without replacement: harder matching algorithm but easier statistical inference
- **Exact match**: perfect covariate balance \mathbf{X}_i for the matched control(s) are the same as the treated unit
 - Infeasible when covariate is continuous / many covariates
- **Coarsened exact matching** (Iacus et al. 2011 Political Anal.)
 - discretize covariates so that you can perform exact match
- **Matching based on a distance**
 - Define a distance measure for any two units: $D(\mathbf{X}_i, \mathbf{X}_j)$
 - Aim to make units within matched sets as close as possible

Matching based on a distance

- Mahalanobis metric matching

$$D(\mathbf{X}_i, \mathbf{X}_j) = \sqrt{(\mathbf{X}_i - \mathbf{X}_j)^\top \widehat{\mathbb{V}}(\mathbf{X})^{-1} (\mathbf{X}_i - \mathbf{X}_j)}$$

$$\widehat{\mathbb{V}}(\mathbf{X}) = \frac{N_t \widehat{\Sigma}_t + N_c \widehat{\Sigma}_c}{N_t + N_c}, \widehat{\Sigma}_t \text{ and } \widehat{\Sigma}_c \text{ are sample covariance matrices for the treated and control}$$

- Propensity score matching

$$D(\mathbf{X}_i, \mathbf{X}_j) = \left| \ln \left(\frac{\hat{e}(\mathbf{X}_i)}{1 - \hat{e}(\mathbf{X}_i)} \right) - \ln \left(\frac{\hat{e}(\mathbf{X}_j)}{1 - \hat{e}(\mathbf{X}_j)} \right) \right|$$

- Hybrid matching methods

- Ensure exact matching in some key covariates: sex
- First stratify units by key covariates, match within each strata using distance-based matching

Matching based on a distance

Nearest-neighbor (NN) matching:

- Define \mathcal{M}_i^c as the set of indices of M closest control units

$$\mathcal{M}_i^c = \left\{ j: W_j = 0, \sum_{l|W_l=0} 1_{\{D(\mathbf{X}_i, \mathbf{X}_j) \leq D(\mathbf{X}_i, \mathbf{X}_l)\}} \leq M \right\}$$

- Matching with replacement

Greedy algorithm

- Define an order of the treated units
- Match M control units with the shortest distance, set them aside, and repeat
- match most difficult units first: order treated units in a descending order of $\hat{e}(\mathbf{X}_i)$

Optimal matching

- $D: N_t \times N_c$ bipartite matrix of pairwise distance or a cost matrix
- Select N_t elements of D such that there is only one element in each row and one element in each column and the sum of pairwise distances is minimized
- Hungarian algorithm

A simple illustrative example

- Consider 7 units from the Barbiturate exposure data
- Propensity score estimated as described in last lecture

- Matching based on the linearized propensity score

$$\hat{l}(\mathbf{X}_i) = \ln \left(\frac{\hat{e}(\mathbf{X}_i)}{1 - \hat{e}(\mathbf{X}_i)} \right)$$

- Treated unit 1 matched with control unit 5
- Treated unit 2 matched with control unit 3

- NN, greedy algorithm and optimal matching result in the same matched sets here

Unit	W_i	$\hat{e}(\mathbf{X}_i)$	$\hat{l}(\mathbf{X}_i)$
1	1	0.577	0.310
2	1	0.032	-3.398
3	0	0.136	-1.846
4	0	0.003	-5.913
5	0	0.310	-0.798
6	0	0.000	-9.424
7	0	0.262	-1.033

Further restrictions on the matched sets

- Rejecting matches of poor quality

- For some units, even the closest match may not be close enough
- Drop treated units if it's hard to find a good match. E.x., drop i if

$$D(\mathbf{X}_i, \mathbf{X}_j) > d_{\max} = 0.1$$

- Often eliminate only treated units with propensity score very close to 1

- How to determine M ?

- $M = 1$
- Matching with Caliper: assign to each treated unit all controls that are within some distance (caliper) of that treated unit
 - Keep all controls j satisfying $D(\mathbf{X}_i, \mathbf{X}_j) \leq d_{\text{cal}}$
 - Can use greedy algorithm
 - Optimal matching: define $D_{ij} = \infty$ if $D_{ij} > d_{\text{cal}}$
- M increases with sample size
- Smaller M , smaller bias but larger variance; larger M , larger bias but smaller variance

Check covariate balancing after matching

- Statistics we can use to assess the balancing of a particular covariate
 - **Standardized mean difference** (also called the normalized difference, not the t-statistics)

$$\Delta_{ct} = \frac{\frac{1}{N_t} \sum_{i=1}^N W_i \left(X_{ik} - \frac{1}{|\mathcal{M}_i^c|} \sum_{i' \in \mathcal{M}_i^c} X_{i'k} \right)}{\sqrt{s_t^2}}$$

May compare Δ_{ct} with 0.1

- Before matching, we may calculate the denominator of Standardized mean difference as $\sqrt{(s_t^2 + s_c^2)/2}$
- **Log ratio of the sample variances** $\Gamma_{ct} = \ln(s_t) - \ln(s_c)$
- Comparing the distribution function in the treated group and control group
 - Empirical cdf: $\hat{F}_c(x) = \frac{1}{N_c} \sum_{i:W_i=0} \mathbf{1}_{X_i \leq x}$, and $\hat{F}_t(x) = \frac{1}{N_t} \sum_{i:W_i=1} \mathbf{1}_{X_i \leq x}$
 - Proportion of treated units outside of the 2.5% and 97.5% quantiles of the control distribution

$$\hat{\pi}_t^{0.05} = \left(1 - \left(\hat{F}_t \left(\hat{F}_c^{-1}(0.975) \right) \right) + \hat{F}_t \left(\hat{F}_c^{-1}(0.025) \right) \right)$$

Love plot

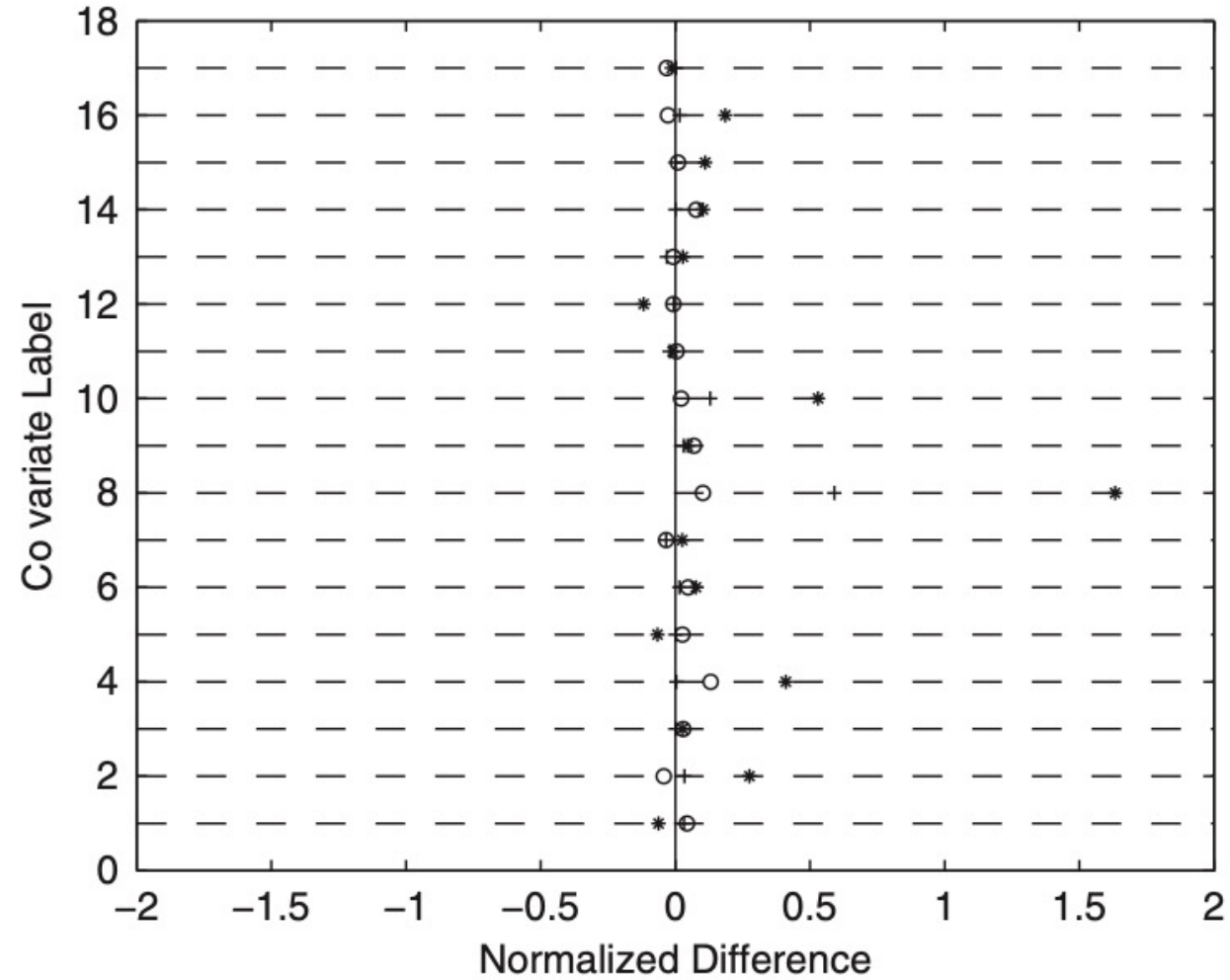


Figure 15.2. Covariate balance before (*) and after (+) lps and after Mahalanobis (o) matching, for the Reinisch barbiturate data

Matching results on the Barbiturate data

Before matching: many control units have low propensity score

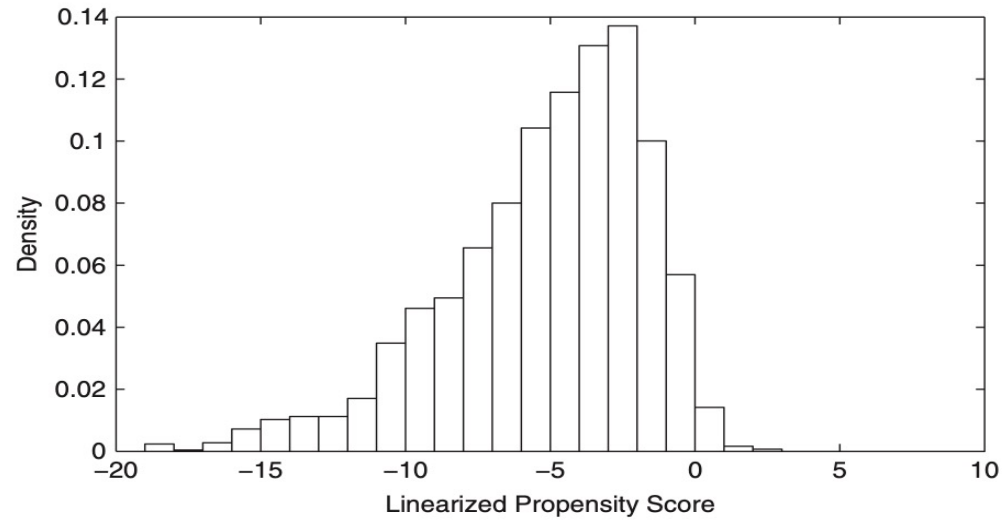


Figure 15.1a. Histogram-based estimate of the distribution of linearized propensity score for control group, for Reinisch barbiturate data

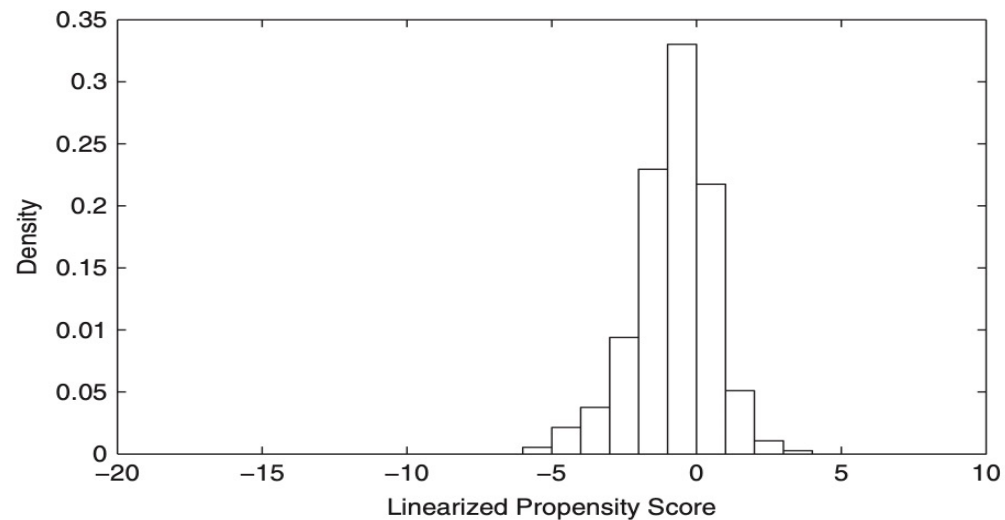


Figure 15.1b. Histogram-based estimate of the distribution of linearized propensity score for treatment group, for Reinisch barbiturate data

Matching results on the Barbiturate data

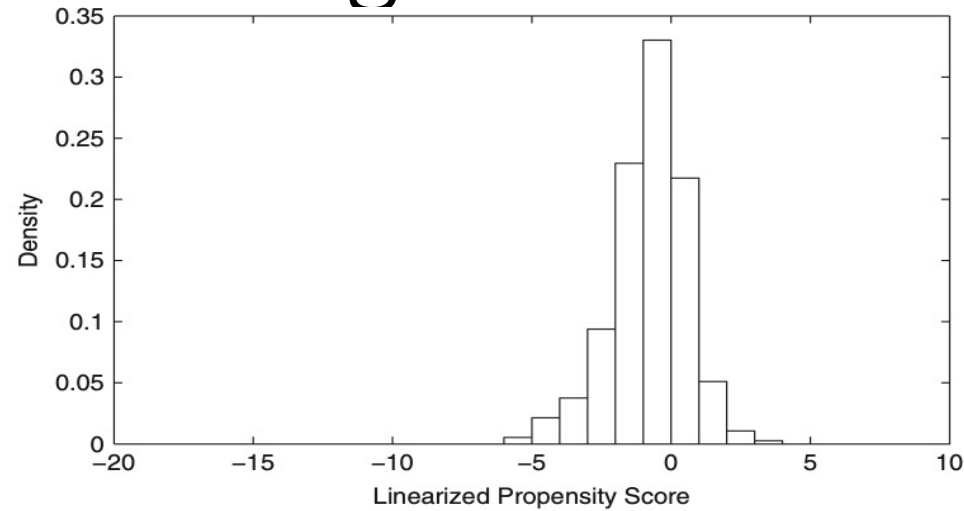


Figure 15.3a. Histogram-based estimate of the distribution of linearized propensity score after lps matching for the treatment group, for the Reinisch barbiturate data

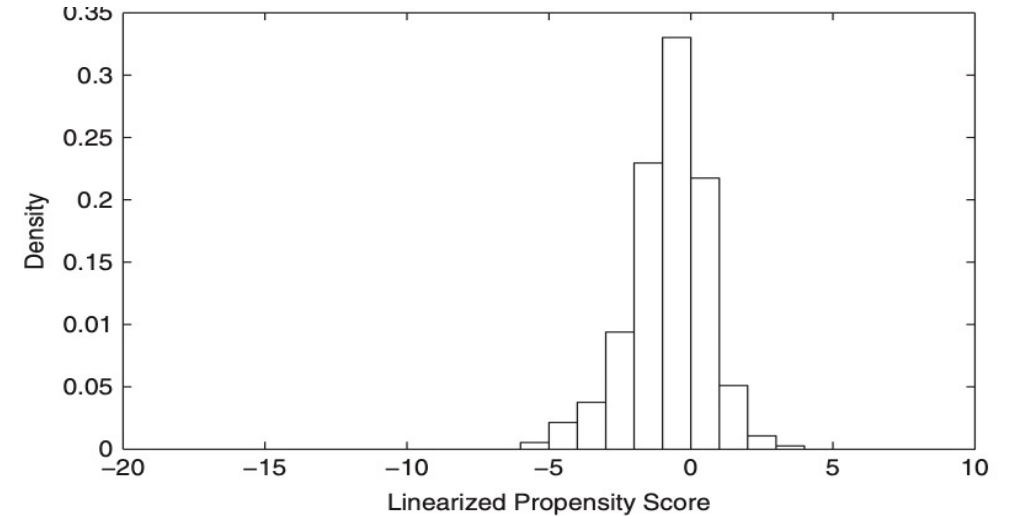


Figure 15.4a. Histogram-based estimate of the distribution of linearized propensity score after Mahalanobis matching for the treatment group, for the Reinisch barbiturate data

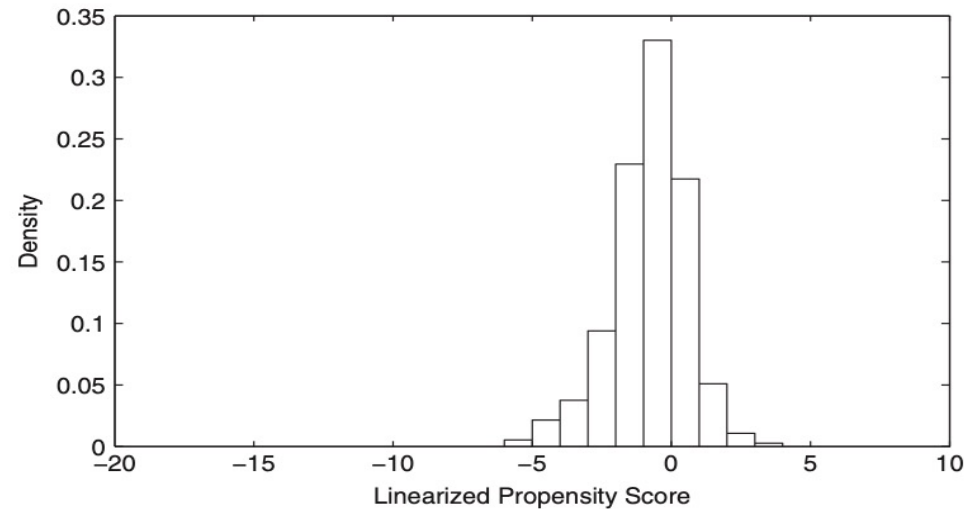


Figure 15.3b. Histogram-based estimate of the distribution of linearized propensity score after lps matching for the control group, for the Reinisch barbiturate data

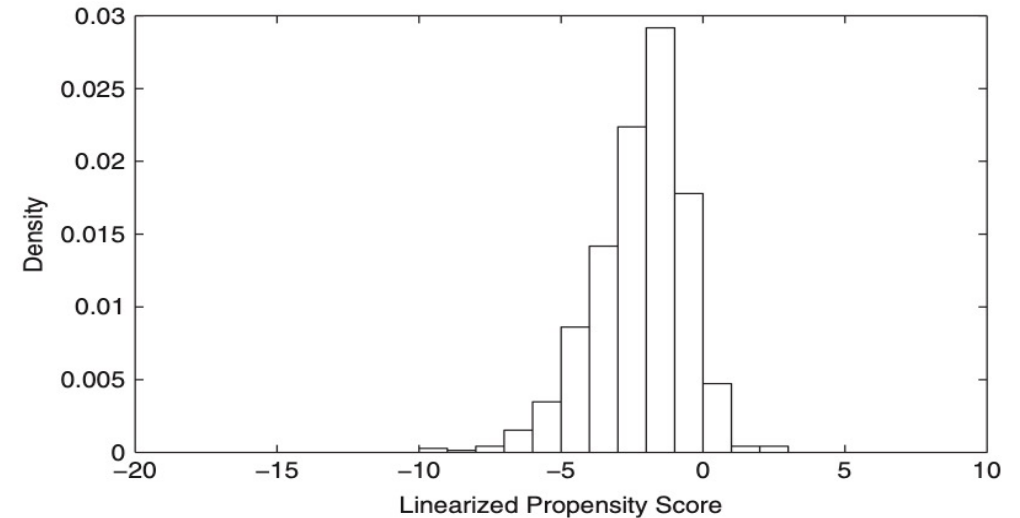


Figure 15.4b. Histogram-based estimate of the distribution of linearized propensity score after Mahalanobis matching for the control group, for the Reinisch barbiturate data

Matching results on the Barbiturate data

Table 15.3. Between Treated and Control Units before and after Matching for the Reinisch Barbiturate Data

	Full Sample				Matched Samples							
	Nor Dif	Log Rat of STD	$1 - \pi^{0.05}$		Nor Dif	Log Rat of STD	$1 - \pi^{0.05}$		Nor Dif	Log Rat of STD	$1 - \pi^{0.05}$	
			Controls	Treated			Controls	Treated			Controls	Treated
sex	-0.01	0.00	1.00	1.00	0.00	-0.00	1.00	1.00	-0.03	0.00	1.00	1.00
antih	0.19	0.20	1.00	1.00	0.02	0.01	1.00	1.00	-0.03	-0.02	1.00	1.00
hormone	0.11	0.43	1.00	0.97	0.00	0.00	1.00	1.00	0.01	0.03	1.00	0.97
chemo	0.10	0.14	1.00	1.00	0.00	0.00	1.00	1.00	0.08	0.10	1.00	1.00
cage	0.03	-0.04	0.93	0.97	-0.03	0.03	0.96	0.95	-0.01	-0.00	0.95	0.95
cigar	-0.12	0.00	1.00	1.00	-0.01	-0.00	1.00	1.00	-0.01	-0.00	1.00	1.00
lgest	-0.01	-0.17	0.95	0.98	-0.02	0.13	0.98	0.97	0.00	0.01	0.98	0.97
lmotage	0.53	0.00	0.93	0.93	0.13	0.02	0.97	0.95	0.02	-0.01	0.95	0.97
lpbc415	0.05	0.06	0.99	0.97	0.03	0.06	0.98	0.99	0.07	-0.06	0.99	0.97
lpbc420	1.63	-0.55	0.52	0.72	0.59	-0.01	0.90	0.86	0.10	0.09	0.96	0.94
motht	0.03	0.03	1.00	1.00	-0.03	0.15	1.00	1.00	-0.03	0.03	1.00	1.00
motwt	0.08	0.02	1.00	1.00	0.02	0.09	1.00	1.00	0.05	-0.02	1.00	1.00
mbirth	-0.07	-0.21	0.97	1.00	0.00	0.00	0.98	0.98	0.03	0.12	0.99	0.98
psydrug	0.41	0.47	1.00	1.00	0.00	0.00	1.00	1.00	0.13	0.09	1.00	1.00
respir	0.03	0.07	1.00	1.00	0.00	0.00	1.00	1.00	0.03	0.07	1.00	1.00
ses	0.28	0.06	1.00	1.00	0.03	0.08	0.99	0.96	-0.04	0.02	0.99	0.96
sib	-0.06	0.00	1.00	1.00	0.03	-0.00	1.00	1.00	0.04	-0.00	1.00	1.00
Multivariate measure	0.43				0.24				0.05			
pscore	1.67	0.62	0.44	0.63	1.33	0.08	0.83	0.82	0.08	0.11	0.96	0.93
linearized pscore	1.65	-0.96	0.44	0.63	0.45	0.11	0.83	0.82	0.02	0.11	0.96	0.93

(corrected typo in textbook)

Better balance of influential covariates of the propensity score using propensity score matching

Five worst matches

Table 15.7. Five Worst Matches for LPS Matching in Terms of LPS Distance, for the Reinisch Barbiturate Data

P-Score		LPS		Dif in LPS
Treated	Control	Treated	Control	
0.79	0.66	1.34	0.64	0.69
0.79	0.66	1.34	0.67	0.68
0.81	0.69	1.45	0.79	0.66
0.81	0.69	1.45	0.80	0.65
0.97	0.94	3.48	2.83	0.64

Not too terrible, do not discard them

How to estimate ATT after matching

- Unless exact matching, under unconfoundedness, the probability of assignment to the treatment is only approximated the same within each matched set
- In practice, one may **ignore** the potential bias, and analyze the datasets as from a pairwise / stratified randomized experiment

$$\hat{\tau}_i^{\text{match}} = Y_i^{\text{obs}} - Y_{m_i^c}^{\text{obs}}, \quad \hat{\tau}_t^{\text{match}} = \frac{1}{N_t} \sum_{i:W_i=1} \hat{\tau}_i^{\text{match}}$$

$$\hat{V} \left(\hat{\tau}_t^{\text{match}} \right) = \frac{1}{N_t(N_t - 1)} \sum_{i:W_i=1} \left(Y_i^{\text{obs}} - Y_{m_i^c}^{\text{obs}} - \hat{\tau}_t^{\text{match}} \right)^2$$

- Another approach is to apply outcome regression on the matched dataset
 - Treat matching is a pre-processing step to improve covariate balancing in the dataset
 - Reduce bias in matching
 - Or we can use regression to only adjust for the potential biases (see later)

The minimum wage data

- An influential study by Card and Krueger (1995)
- The goal is to evaluate the effect of raising the state minimum wage in New Jersey in 1993
- They collected data on employment at fast-food restaurants in New Jersey (treated group) and in neighboring state of Pennsylvania (control group)
- Each unit is a restaurant
- Pre-treatment covariates: initial number of employees, starting wage, average time until first raise, identity of the chain
- Outcome: number of employees after the raise in the minimum wage

The minimum wage data

Table 18.1. *The Card-Krueger New Jersey and Pennsylvania Minimum Wage Data* (corrected typo)

	$(N = 347)$		$(N_c = 68)$		$(N_t = 279)$		Nor Dif	Log Ratio of STD	$\pi^{0.05}$	
	Mean	(S.D.)	Mean	(S.D.)	Mean	(S.D.)			Controls	Treated
initial empl	17.84	(9.62)	20.17	(11.96)	17.27	(8.89)	-0.28	-0.30	0.10	0.03
burger king	0.42	(0.49)	0.43	(0.50)	0.42	(0.49)	-0.02	-0.01	0.00	0.00
kfc	0.19	(0.40)	0.13	(0.34)	0.21	(0.41)	0.20	0.17	0.00	0.00
roys	0.25	(0.43)	0.25	(0.44)	0.25	(0.43)	0.00	-0.00	0.00	0.00
wendys	0.14	(0.35)	0.19	(0.40)	0.13	(0.33)	-0.18	-0.18	0.00	0.00
initial wage	4.61	(0.34)	4.62	(0.35)	4.60	(0.34)	-0.05	-0.02	0.03	0.01
time until raise	17.96	(11.01)	19.05	(13.46)	17.69	(10.34)	-0.11	-0.26	0.10	0.03
pscore	0.80	(0.05)	0.79	(0.06)	0.81	(0.04)	0.28	-0.35	0.10	0.03
final empl	17.37	(8.39)	17.54	(7.73)	17.32	(8.55)				

The minimum wage data

Estimated propensity score model:

Higher initial employment, lower propensity score

$$\hat{l}(X_i) = 1.93 - 0.03 \times \text{initial empl}$$

Table 18.2. *Estimated Parameters of Propensity Score for the Card-Krueger New Jersey and Pennsylvania Minimum Wage Data*

Variable	Est	(s. e.)	t-Stat
Intercept	1.93	(0.14)	14.05
Linear terms			
initial empl	-0.03	(0.01)	-2.17

The minimum wage data on 20 units

Unit i	State W_i	chain X_{i1}	initial empl X_{i2}	final empl Y_i^{obs}
1	NJ	BK	22.5	40.0
2	NJ	KFC	14.0	12.5
3	NJ	BK	37.5	20.0
4	NJ	KFC	9.0	3.5
5	NJ	KFC	8.0	5.5
6	PA	BK	10.5	15.0
7	PA	KFC	13.8	17.0
8	PA	KFC	8.5	10.5
9	PA	BK	25.5	18.5
10	PA	BK	17.0	12.5
11	PA	BK	20.0	19.5
12	PA	BK	13.5	21.0
13	PA	BK	19.0	11.0
14	PA	BK	12.0	17.0
15	PA	BK	32.5	22.5
16	PA	BK	16.0	20.0
17	PA	KFC	11.0	14.0
18	PA	KFC	4.5	6.5
19	PA	BK	12.5	31.5
20	PA	BK	8.0	8.0

- Matching order:
if we rank based on $\hat{e}(X_i)$: 5, 4, 2, 1, 3
- Matching metric:
 - Only based on $\hat{l}(X_i)$: 20, 8, 7, 11, 15
 - If we want exact match on the chain brand
5 <-> 8, 4 <-> 17, 2 <-> 7, 1 <-> 11, 3 <-> 15
 - If we want to match on Mahalanobis distance, can code the restaurant brand by 0/1 indicators, then 5 <-> 20, 4 <-> 8

The minimum wage data on 20 units

i	m_i^c	Y_i^{obs}	$Y_{m_i^c}^{\text{obs}}$	$\hat{\tau}_i^{\text{match}}$	i	m_i^c	Y_i^{obs}	$Y_{m_i^c}^{\text{obs}}$	$\hat{\tau}_i^{\text{match}}$
1	11	40.0	19.5	20.5	1	11	40.0	19.5	20.5
2	7	12.5	17	-4.5	2	7	12.5	17.0	-4.5
3	15	20.0	22.5	-2.5	3	15	20.0	22.5	-2.5
4	8	3.5	10.5	-7	4	17	3.5	14	-10.5
5	20	5.5	8.0	-2.5	5	8	5.5	10.5	-5
$\hat{\tau}_t^{\text{match}}$				+0.8	$\hat{\tau}_t^{\text{match}}$				-0.4
$\hat{V}(\hat{\tau}_t^{\text{match}})$				5.0 ²	$\hat{V}(\hat{\tau}_t^{\text{match}})$				5.4 ²

The bias of matching estimators

- Individual treatment effect is estimated with a bias due to matching discrepancy

$$\begin{aligned}\mathbb{E}_{\text{sp}} \left[\hat{\tau}_i^{\text{match}} \mid W_i = 1, X_i, X_{m_i^c} \right] &= \mathbb{E}_{\text{sp}} \left[Y_i(1) - Y_{m_i^c}(0) \mid X_i, X_{m_i^c} \right] = \mu_t(X_i) - \mu_c(X_{m_i^c}) \\ &= \tau(X_i) + (\mu_c(X_i) - \mu_c(X_{m_i^c})).\end{aligned}$$

We refer to the last term of this expression,

$$B_i = \mu_c(X_i) - \mu_c(X_{m_i^c}),$$

as the *unit-level bias* of the matching estimator.

- If we can have estimates of B_i , then we can potentially correct for the biases
- We can obtain the estimates of B_i by outcome regression: only need an estimate $\hat{\mu}_0(\mathbf{X}_i)$

$$\hat{Y}_i(0) = Y_{m_i^c}(0) + \hat{B}_i$$

Three types of regression

- Regression on the differences

$$Y_i^{\text{obs}} - Y_{m_i^c}^{\text{obs}} = \tau + (X_i - X_{m_i^c}) \beta_d + v_i = \tau + D_i \beta_d + v_i$$

$$\hat{Y}_i(0) = Y_{m_i^c}(0) + \hat{B}_i = Y_{m_i^c}(0) + (X_i - X_{m_i^c}) \hat{\beta}_d$$

- Regression only on the matched control

$$Y_{m_i^c} = \alpha_c + X_{m_i^c} \beta_c + v_{ci}$$

$$\hat{Y}_i(0) = Y_{m_i^c}(0) + (X_i - X_{m_i^c}) \hat{\beta}_c$$

- Regression on both the treated and the matched controls (pooled sample)

$$\tilde{Y}_i = \alpha_p + \tau_p \cdot \tilde{W}_i + \tilde{X}_i \beta_p + v_i$$

$$\hat{Y}_i(0) = Y_{m_i^c}(0) + (X_i - X_{m_i^c}) \hat{\beta}_p$$

- These methods differ in their robustness to model assumptions and efficiency

Results on the 20 units

	Difference Regression (Approach #1)	Control Regression (Approach #2)	Pooled Regression (Approach #3)
Regression coefficients			
Intercept	-1.30	4.21	12.01
Treatment indicator	—	—	1.63
Restaurant chain	-1.20	2.65	-7.32
Initial employment	1.43	0.62	0.39

- Different regression methods differ a lot because small sample size
- In real data, they are typically similar

Results on the 20 units

Results from first bias-adjustment approach

i	m_i^c	$Y_i(1)$	$Y_{m_i^c}(0)$	$X_{i,1}$	$X_{i,2}$	$X_{m_i^c,1}$	$X_{m_i^c,2}$	$D_{i,1}$	$D_{i,2}$	$D_i \hat{\beta}_d^T$	$\hat{Y}_i(0)$
1	11	40.0	19.5	0	22.5	0	20.0	0	2.5	3.6	23.1
2	7	12.5	17.0	1	14.0	1	13.8	0	0.2	0.3	17.3
3	15	20.0	22.5	0	37.5	0	32.5	0	5.0	7.1	29.6
4	8	3.5	10.5	1	9.0	1	8.5	0	0.5	0.7	11.2
5	20	5.5	8.0	1	8.0	0	8.0	1	0	-1.2	6.8

$\hat{\tau}_t^{\text{match}} = +0.8$ $\hat{\tau}_t^{\text{adj}} = -1.3$

Results on the full minimum wage data

Table 18.12. *Bias-Adjusted Matching Estimators for the Card-Krueger New Jersey and Pennsylvania Minimum Wage Data*

Variable	Regression Coefficients		
	Difference Regression	Control Regression	Pooled Regression
Initial employment	0.50	0.12	0.35
Restaurant chain:			
KFC	-23.27	4.05	2.03
Roys	-	-3.62	-3.03
Wendys	-	-3.23	-2.00
Starting wage	-3.20	7.07	2.13
Time till first raise	-0.01	0.12	0.07
$\hat{\tau}_t^{\text{adj}}$	0.51	0.71	0.79

Three type of regression adjustment do not differ that much

Results on the full minimum wage data

Estimand	Method	M	Metric	Estimate
	OLS, no covariates			-0.22
New Jersey	OLS, covariates adjusted			1.35
New Jersey	Match	1	Mahalanobis	0.89
New Jersey	Match	4	Mahalanobis	1.01
New Jersey	Match	1	Euclidean	0.93
New Jersey	Match	1	Exact on Chain, Mahal. on Others	0.92
Pennsylvania	Match	1	Mahalanobis	0.63
All	Match	1	Mahalanobis	0.84
New Jersey	Bias adj, dif regress	1	Mahalanobis	0.51
New Jersey	Bias adj, control regress	1	Mahalanobis	0.71
New Jersey	Bias adj, pooled regress	1	Mahalanobis	0.79