Lecture 4
Fisher's exact test: R example and case study

Outline

- Computing Fisher's exact p-value and CI with R
- A case study using Fisher's sharp null and exact p-values

Case study: the California alphabet lottery

[Randomization inference with natural experiments: An analysis of ballot effects in the 2003 California recall election. *Journal of the American statistical association, 2006]*

Problem background

- In the 2000 U.S. national election, George W. Bush became President by winning 537 more votes than Al Gore in Florida.
- This unusually close election result served as a reminder that the manner in which elections are administered can change outcomes.

- This paper studied the causal effect of the page placement of candidates in the 2003 California recall election
- dataset was collected by *The New York Times* in 2003 (not publicly available)

Case study: the California alphabet lottery

[Randomization inference with natural experiments: An analysis of ballot effects in the 2003 California recall election. *Journal of the American statistical association, 2006]*

Problem background

- Recall results

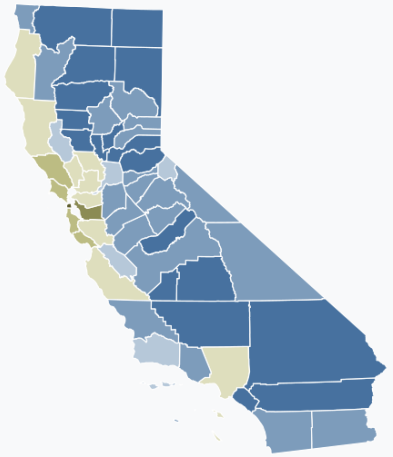
https://en.wikipedia.org/wiki/2003_California_gubernatorial_recall_election

Vote on recall

Shall Gray Davis be recalled (removed) from the office of Governor?

Results

Response	Votes	%
✓ **Yes**	**4,976,274**	**55.39%**
✗ **No**	4,007,783	44.61%
Valid votes	8,984,057	95.44%
Invalid or blank votes	429,431	4.56%
Total votes	**9,413,488**	**100.00%**
Registered voters/turnout	15,380,536	61.2%



County results

Yes: 50–60% 60–70% 70–80%

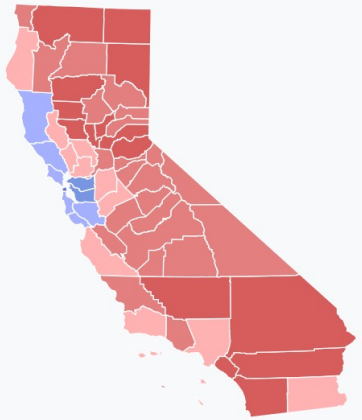
No: 50–60% 60–70% 70–80% 80–90%

Replacement candidates

If Davis is recalled, who should replace him as governor?

Turnout 61.20%

Candidate	**Arnold Schwarzenegger**	Cruz Bustamante	Tom McClintock
Party	Republican	Democratic	Republican
Popular vote	**4,206,284**	2,724,874	1,161,287
Percentage	**48.6%**	31.5%	13.4%



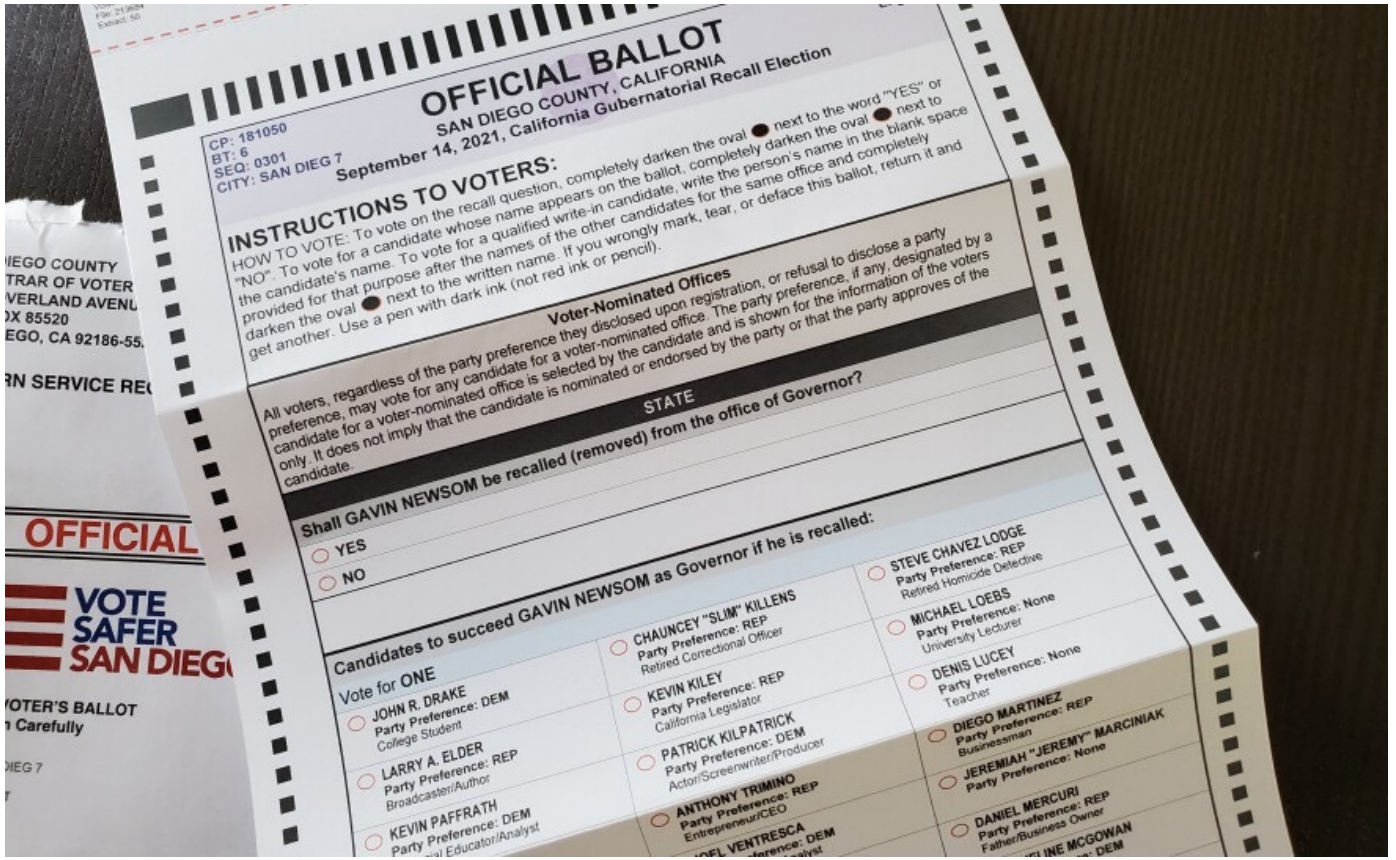
County results

Schwarzenegger: 40–50% 50–60% 60–70%

Bustamante: 40–50% 50–60% 60–70%

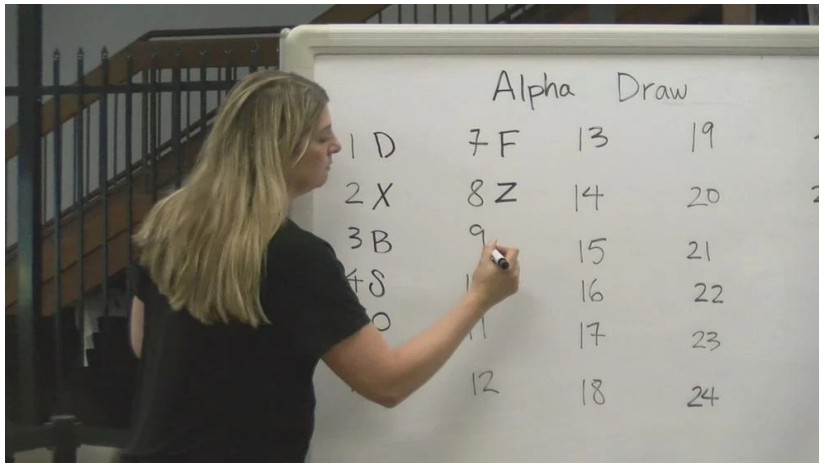
Causal question

- Whether the placement of candidates on ballot have any causal effect on the election result



The randomization-rotation procedure

- Since 1975, California law has mandated that the Secretary of State draw a random alphabet for each election to determine the order of candidates for the first assembly district [California Election Code § 13112 (2003)].
- California law further requires that the candidate order be systematically rotated throughout the remaining assembly districts.



- The procedure
  1. Randomize alphabet
  2. Sort candidates by randomized alphabet
  3. Rotate the candidate order from the first district

For the 2003 recall election, the actual randomized alphabet was

R W Q O J M V A H B S G Z X N T C I E K U P D Y F L

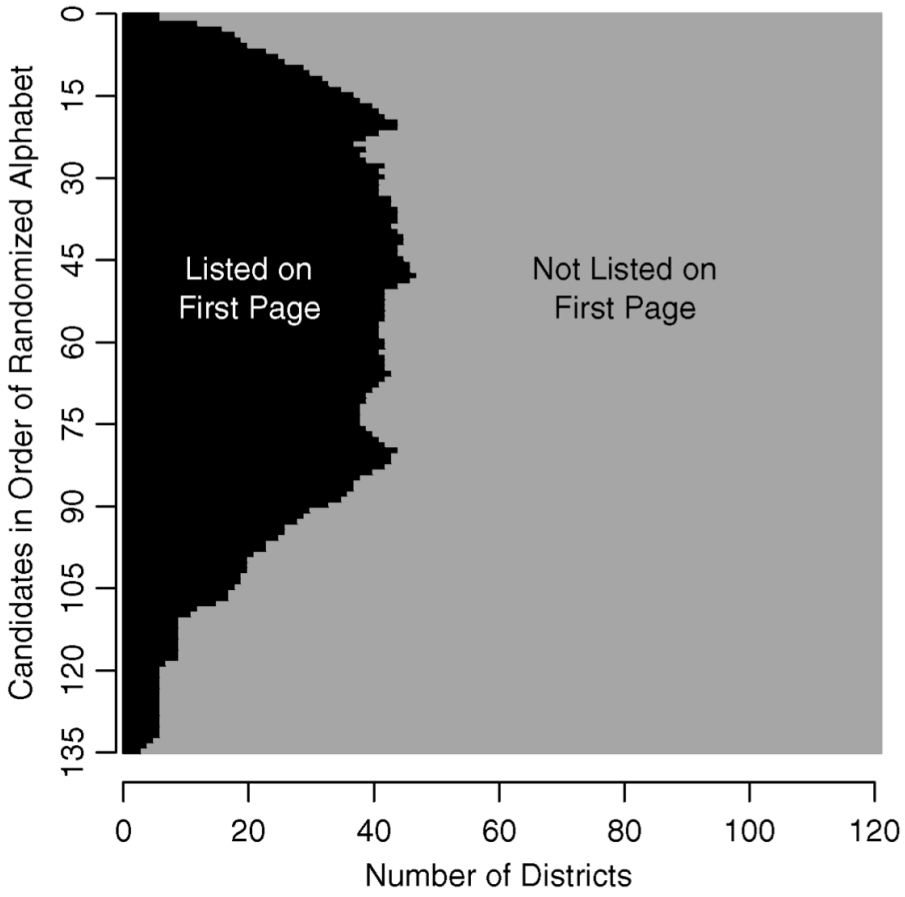
- The ballot order in the first assembly district was determined, starting from Robinson, Roscoe, Ramirez, and so on and proceeding to Lewis and Leonard.
- This candidate order was then rotated throughout the remaining assembly districts.

The randomization-rotation procedure

Challenges analyzing data with the randomization procedure

- an unprecedented total of 135 candidates, page placement is not even across them
- Each of the 58 counties uses a different ballot format with varying numbers of pages, leading to 121 county-district combinations of ballot formats
- interactions across candidates

- The alphabets are randomized, but the 80 assembly districts order are not randomized



No complete randomization of page placement across candidates nor across districts

Comparison across districts or across candidates?

- Use randomization inference
  - Take into consideration the unconventional treatment assignment mechanism
  - Test for Fisher's sharp null of no causal effect on any unit

- Choice 1: comparison across candidates within the same county-district combination
  - What are the units and treatment assignments?
  - What are the potential outcomes?
  - Is the unconfoundedness property satisfied on the treatment assignment mechanism?
  - Is the no interference assumption reasonable?
  - Is the consistency assumption reasonable?
  - Strong heterogeneity across candidates in the potential outcomes.

- Choice 2: Comparison across county-district combination within each candidate
  - Choice made by the researchers

Set up the analysis framework

- Analyze the causal effect of page placement for each of the 135 candidates separately
- Each of 121 county-district combination is a **unit**: $Y_i(0)$ and $Y_i(1)$ for a district i and a particular candidate
- Treatment: $T_i = 1$ if candidate is placed on the first page, $T_i = 0$ otherwise
- Sharp null for a particular candidate: $H_0: Y_i(0) \equiv Y_i(1)$ for all $i = 1, \cdots, 121$

- Test statistics:
  - Sample average treatment effect $W^{\mathrm{D}}(\mathbf{T}) = \frac{\sum_{i=1}^{121} T_i y_i}{N_1} - \frac{\sum_{i=1}^{121}(1-T_i) y_i}{N_0}$
  - Covariate-adjusted test statistics

$$W^{\mathrm{L}}(\mathbf{T}) = (\mathbf{T}^\top \mathbf{M} \mathbf{T})^{-1} \mathbf{T}^\top \mathbf{M} \mathbf{y}, \qquad (4)$$

where $\mathbf{y} = (y_1, y_2, \ldots, y_{121})$, $\mathbf{M} = \mathbf{I} - \mathbf{X}(\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top$, and $\mathbf{X}$ is the matrix of the observed pretreatment covariates.

Set up the analysis framework

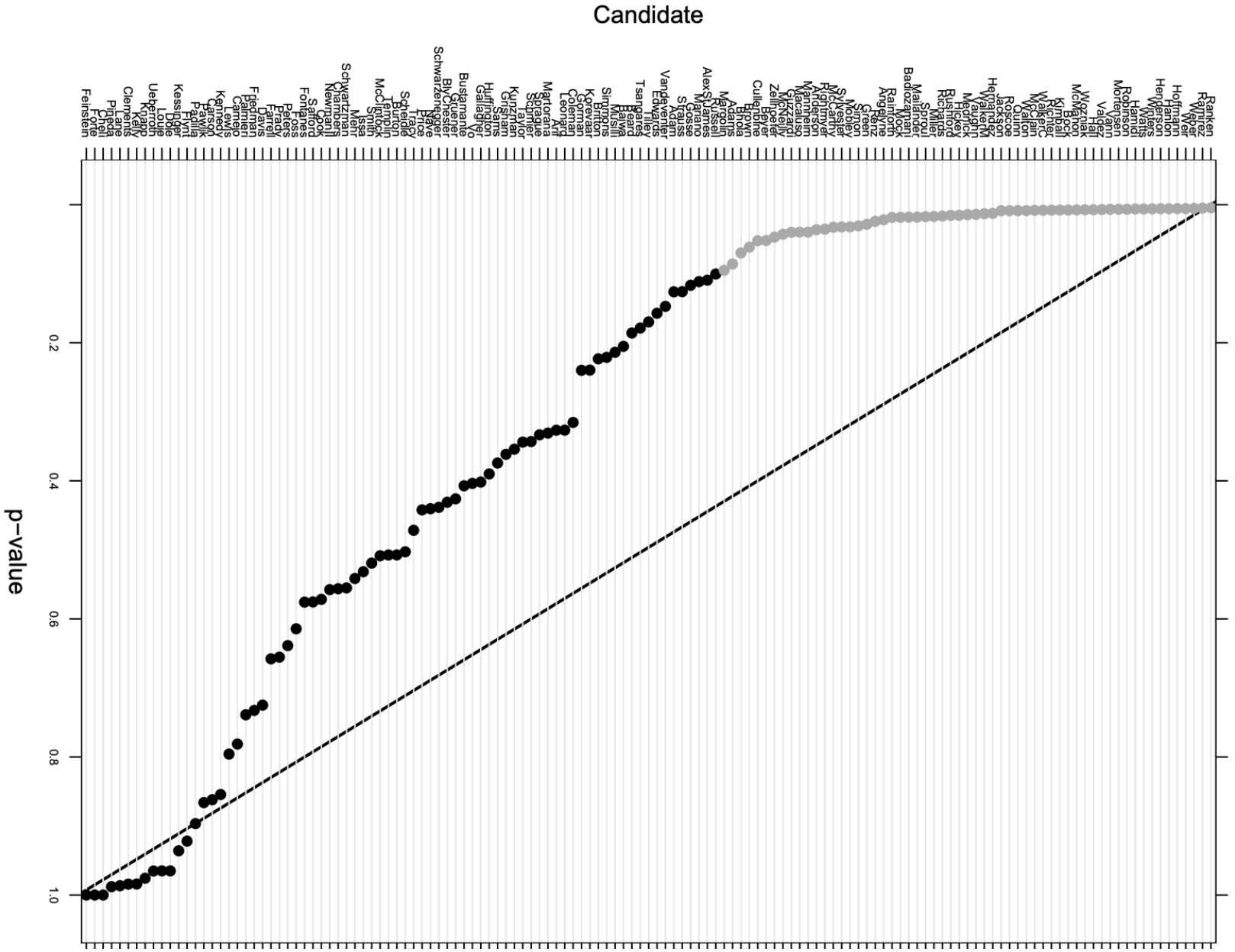
Implicit assumptions

- *Assumption 1* (No interference among units) The potential outcomes of one unit do not depend on the treatment of other units.
  - potential vote shares of a candidate in one district do not depend on the same candidate's ballot placement in another district.
  - Voters usually do not see ballots of other districts and hence are unlikely to be affected by such ballots.

- *Assumption 2* (Known random assignment). Treatment is randomly assigned by a known mechanism. Formally, $p(T_i|Y_i(0), Y_i(1)) = p(T_i)$ is known for each i.
  - Assumes county page formats are independent of the randomized alphabet
  - Number of possible ballot pages is driven primarily by the type of voting technology, should not be designed based on the randomized alphabet result

Distribution of Exact p-values across Candidates

- Authors computed the one-sided p-values
- Reference distribution obtained via Monte Carlo
- Candidates ranked based on their p-values
- If the sharp null is true, these p-values should all be uniformly distributed



Confidence intervals under the constant additive effect model

- For each candidate, we assume $Y_i(0) - Y_i(1) \equiv \tau_0$ across all republican / democratic districts
- We construct confidence intervals by inverting the Fisher's randomization tests at a range of τ_0 values