Topics in Causal Inference

STAT41530

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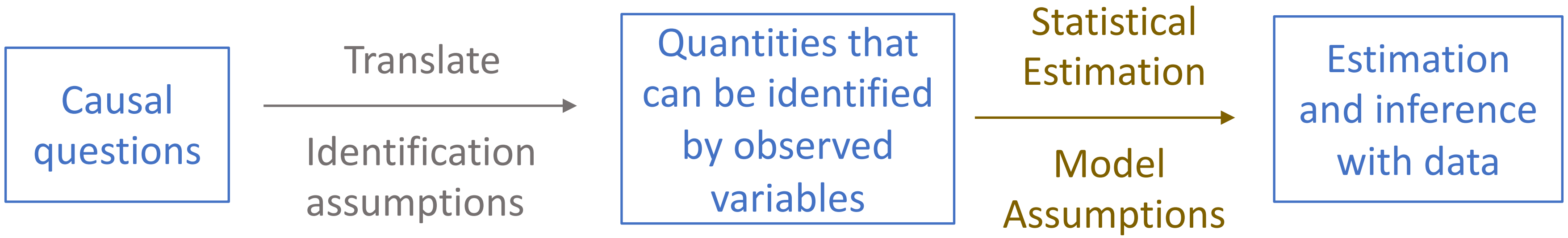
Outline

- Week 1-5: Basic Concepts and methods in causal inference
  - Mostly follow Herman and Robins' book



Chapter 1-4: Potential outcome framework
Chapter 6-9: DAG
Chapter 16: IV
Mediation

Chapter 11-15
Chapter 18

- Week 6-9: Discuss causal papers in genetics / clinical applications

Lecture 1

Topic: Potential outcome framework

- Definition of causal effects
- Randomized experiments
  - Completely randomized experiments
  - Conditional randomizaed experiment

Some slides are borrowed from slides of Prof. Fan Li at Duke Uni.

Causality

- We know what causal effects mean as a human being

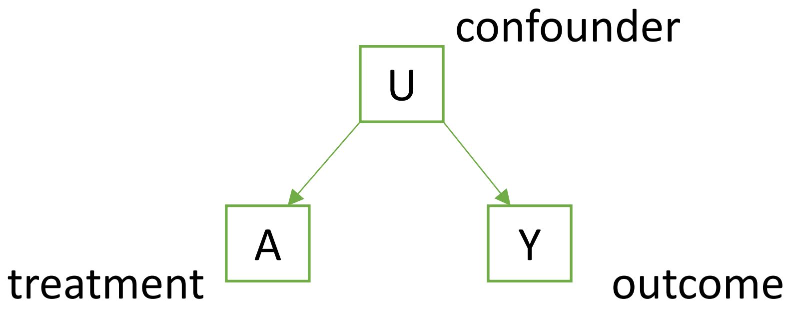
I would rather discover one causal law than be King of Persia.
— Democritus

We have knowledge of a thing only when we have grasped its cause.
— Aristotle, Posterior Analytics

- How to quantitatively define “causal effects” with mathematical notations?

Association ≠ Causation

- Confounding



- Examples of confounding
  - Ice cream consumption and number of people drowned. Confounder: temperature
  - Medical treatment and patient outcome. Confounder: age, sex, other complications
  - Education and income. Confounder: family
  - Confounder can reserve the sign of the correlation between treatment and outcome (Simpson’s paradox, discuss in later slides)

The potential outcome framework
[Neyman (1923), Rubin (1974)]

- $A = 1$ or 0: treatment with two levels (treatment and no treatment)

- For an individual i:

  - $Y_i(1)$: whether he/she survives if receiving treatment

  - $Y_i(0)$: whether he/she survives if not receiving treatment

  Potential outcomes (counterfactuals):
  only one of the potential outcomes can be observed

Causal effect of A on Y for individual i

$$Y_i(1) \neq Y_i(0)$$

- Observed data: $Y_i = Y_i(1)A_i + Y_i(0)(1 - A_i)$

Assumptions in the above notation? (SUTVA)

- Consistency
  - There is only one version of the treatment

    $Y(a)$ needs to be well defined

    -- counterexamples: effect of BMI, specific procedure of a treatment
  - We assume that $Y = Y(A)$

    -- counterexamples: drug effect in a trial v.s. in reality

These two assumptions are also called SUTVA
(Stable Unit Treatment Value Assumption) [Rubins 1978, 1980, 1990]

- No interference

  One individual's outcome is not affected by other individuals'

  -- counterexamples: vaccination, advertising, infectious disease, social networks, agricultural experiments

Average causal effects

- $Y_i(1) \neq Y_i(0)$ impossible to know for every individual
- One quantity that is potentially easiest to identify:

  Average causal effect: $E(Y(1)) \neq E(Y(0))$ for a target population

  - Average treatment effect: $E(Y(1) - Y(0))$
  - Causal risk ratio [for binary outcome]: $\mathrm{P}(Y(1) = 1)/P(Y(0) = 1)$

- There are other quantities of the causal effects that we can quantify, but they need to be functions of the potential outcomes $Y(a)$
- How to identify these quantities from observed Y? ($Y = Y(1)A + Y(0)(1 - A)$)

  Essentially a missing data problem!

Completely Randomized Experiments

- For 30 people in the experiment, flip a coin (not necessarily unbiased) to decide who receives a treatment
- Randomly select 10 people to receive treatment

We have

$$A \perp Y(a) \text{ for all } a$$

Called exchangeability / ignorability

Identify average causal effects

$$
\begin{aligned}
&\mathbb{E}\left[Y(a)\right] \\
=&\mathbb{E}\left[Y(a) \mid A=a\right] \quad \text{Exchangeability} \\
=&\mathbb{E}\left[Y(A) \mid A=a\right] \\
=&\mathbb{E}\left[Y \mid A=a\right] \quad \text{Consistency}
\end{aligned}
$$

- We are considering an ideal randomized experiment
- What might not be ideal in practice?
  - Adhersive to assignment
  - Censoring / lost to follow-up
  - Multiple versions of assignment
  - Unblinded experiment (placebo effect)

Conditionally randomized experiments

- L: severity of the heart disease ($L = 1$ if severe)
  - $L = 1$: randomly assign treatment to 75% of individuals
  - $L = 0$: randomly assign treatments to 50% of individuals

Conditional exchangeability

$$A \perp Y(a) \mid L \text{ for all } a$$

In each subgroup of L, run a completely randomized experiment

Average causal effects are still identifiable

$$
\begin{aligned}
& \mathbb{E}\left[Y(a)\right] \\
=& \mathbb{E}\left[\mathbb{E}\left[Y(a) \mid L\right]\right] \\
=& \mathbb{E}\left[\mathbb{E}\left[Y(a) \mid L, A=a\right]\right] \quad \text{Conditional exchangeability} \\
=& \mathbb{E}\left[\mathbb{E}\left[Y(A) \mid L, A=a\right]\right] \\
=& \mathbb{E}\left[\mathbb{E}\left[Y \mid L, A=a\right]\right] \quad \text{Consistency}
\end{aligned}
$$

$$\mathbb{E}\left[\mathbb{E}\left[Y \mid L, A = a\right]\right] \neq \mathbb{E}\left[Y \mid a\right]$$

Average causal effect

$$\mathbb{E}\left[\mathbb{E}\left[Y \mid L, A = a\right]\right] = \sum_{l} \mathbb{E}\left[Y \mid L = l, A = a\right] P(L = l)$$

Does not have a causal interpretation

$$\mathbb{E}\left[Y \mid A = a\right] = \sum_{l} \mathbb{E}\left[Y \mid L = l, A = a\right] P(L = l \mid A = a)$$

Distribution of L conditional on different values of A can be different
(L is a confounder)

Simpson's paradox: kidney stone treatment

- Compare the success rates of two treatment of kidney stores
- Treatment A: open surgery; treatment B: small puctures

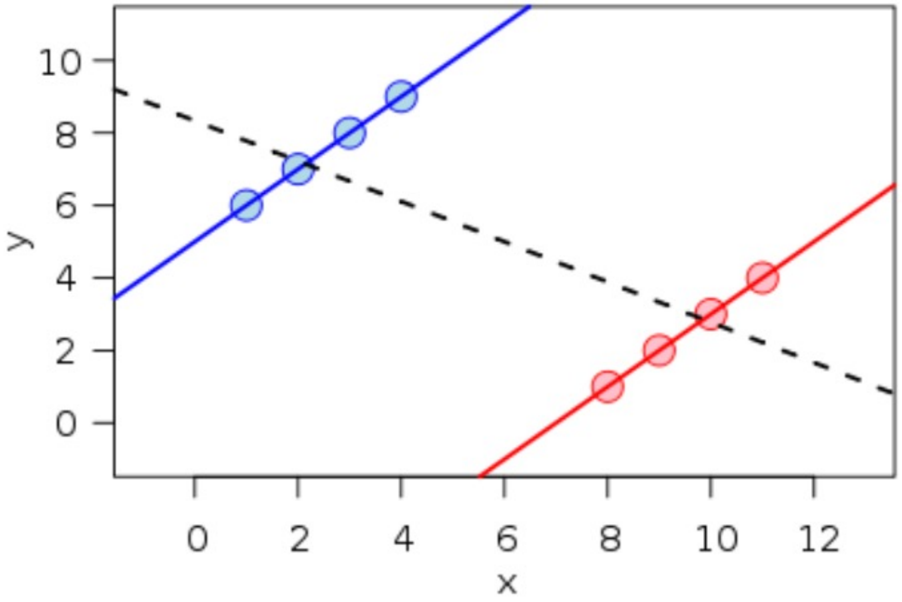
	Treatment A	Treatment B
Small stones	**93%** (81/87)	87% (234/270)
Large stones	**73%** (192/263)	69% (55/80)
Both	78% (273/350)	**83%** (289/350)

- What is the confounder here? Severity of the case

Simpson's paradox or Yule-Simpson effect

(K Pearson et al. 1899; Yule 1903; Simpson 1951)

- Simpson's paradox: a trend appears in different groups of data but disappears or reverses when these groups are combined



- Another well-known example is the Berkeley admission gender bias (Bickel et al., Science, 1976)

Standardization

$$\mathbb{E}\left[Y(a)\right] = \mathbb{E}\left[\mathbb{E}\left[Y \mid L, A = a\right]\right] = \sum_{l} \mathbb{E}\left[Y \mid L = l, A = a\right] P(L = l)$$

Table 2.2

	L	A	Y
Rheia	0	0	0
Kronos	0	0	1
Demeter	0	0	0
Hades	0	0	0
Hestia	0	1	0
Poseidon	0	1	0
Hera	0	1	0
Zeus	0	1	1
Artemis	1	0	1
Apollo	1	0	1
Leto	1	0	0
Ares	1	1	1
Athena	1	1	1
Hephaestus	1	1	1
Aphrodite	1	1	1
Cyclope	1	1	1
Persephone	1	1	1
Hermes	1	1	0
Hebe	1	1	0
Dionysus	1	1	0

$$\mathbb{P}\left[L = 1\right] = \frac{12}{20} = \frac{3}{5}$$

$$\mathbb{P}\left[L = 0\right] = \frac{8}{20} = \frac{2}{5}$$

$$\mathbb{E}\left[Y \mid A = 1, L = 1\right] = \frac{2}{3}$$

$$\mathbb{E}\left[Y \mid A = 1, L = 0\right] = \frac{1}{4}$$

$$\mathbb{E}\left[Y(1)\right] = \frac{3}{5} \times \frac{2}{3} + \frac{2}{5} \times \frac{1}{4} = 0.5$$

Inverse probability weighting (IPW)

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Rheia	0	0	0
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Dionysus	1	1	0

$$\begin{aligned}\mathbb{E}[Y(a)] &= \mathbb{E}[\mathbb{E}[Y \mid L, A=a]] \\ &= \mathbb{E}\left[\frac{\mathbb{E}[Y 1_{A=a} \mid L]}{\mathbb{P}[A=a \mid L]}\right] \quad \text{(conditional exchangeability)} \\ &= \mathbb{E}\left[\frac{1_{A=a}}{\mathbb{P}[A=a \mid L]} Y\right] = \mathbb{E}[W_a Y]\end{aligned}$$

$$\mathbb{P}[A=1 \mid L=1] = \frac{3}{4}$$

$$\mathbb{P}[A=1 \mid L=0] = \frac{1}{2}$$

$$\mathbb{E}[Y(1)] = \frac{1}{20}(2 + 4/3 \times 6) = 0.5$$

Propensity score: $e(L) = \mathbb{P}[A=1 \mid L]$

Implicit Assumption

For standardization and IPW we implicitly need assumptions:

1. Discrete A (can be generalized)
2. Positivity / Overlapping

$$P[A = a \mid L] > 0$$

for all l where $P[L = l] > 0$ in the target population (population of interest).

- Intuition: if we assign A = 1 to all patients under severe condition, then there is no information from the data to identify $P[Y(0) | L = 1]$